Exercise 8

For the limit

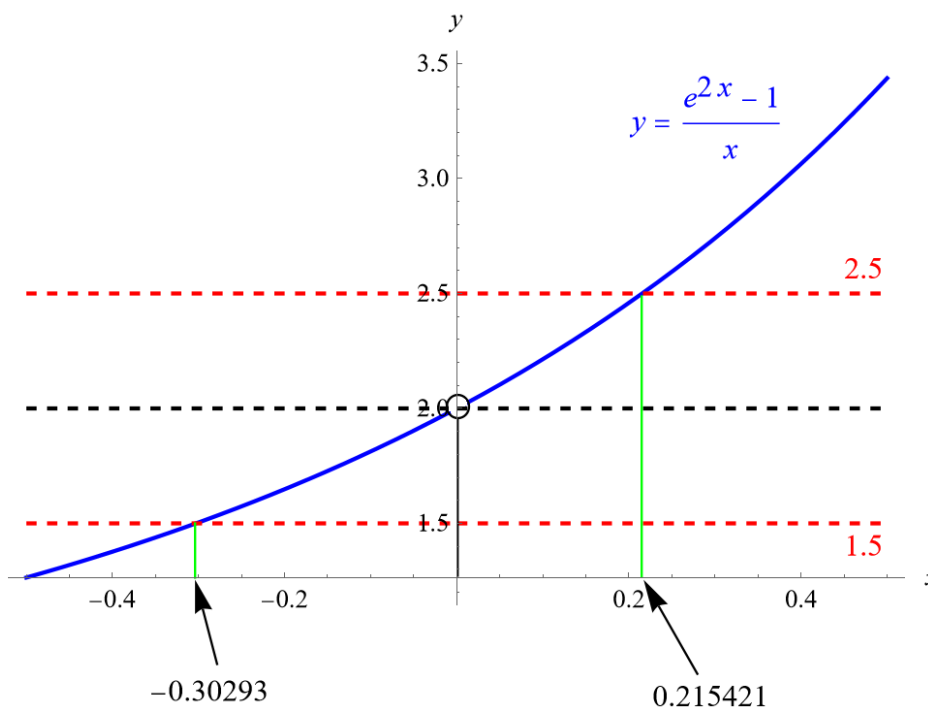$$\lim_{x \to 0} \frac{e^{2x} - 1}{x} = 2$$

illustrate Definition 2 by finding values of δ that correspond to $\varepsilon = 0.5$ and $\varepsilon = 0.1$.

Solution

For $\varepsilon = 0.5$, Definition 2 says that this limit is equivalent to

$$\text{if} \quad 0 < |x - 0| < \delta \quad \text{then} \quad \left| \frac{e^{2x} - 1}{x} - 2 \right| < 0.5$$
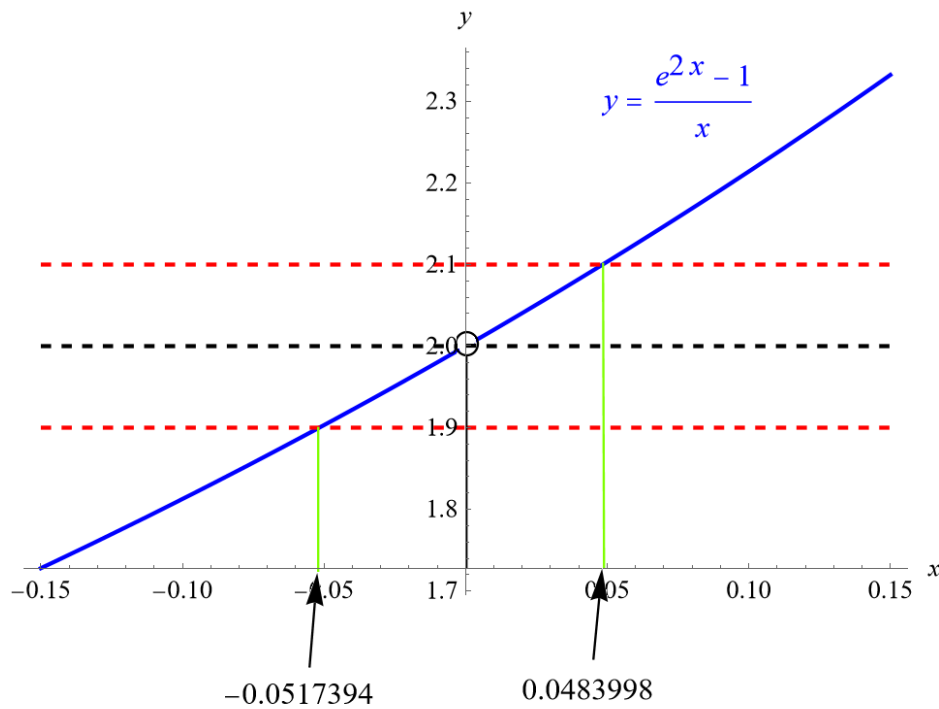
for some positive δ.

As long as δ is less than about $0.215421 - 0 \approx 0.215421$, the distance from 2 on the y-axis will be less than 0.5.

For $\varepsilon = 0.1$, Definition 2 says that this limit is equivalent to

$$\text{if} \qquad 0 < |x - 0| < \delta \qquad \text{then} \qquad \left| \frac{e^{2x} - 1}{x} - 2 \right| < 0.1$$

for some positive δ.

As long as δ is less than about $0.0483998 - 0 \approx 0.0483998$, the distance from 2 on the y-axis will be less than 0.1.